An Application of Karman's Energy Transfer Process in Hydromagnetic Turbulence

G. C. Pramanik

Vill. Jhokurkul, P.O. Jafully-Bibichak, Via — Mechada, Dist. Midnapur, West Bengal, India

Z. Naturforsch. **34a**, 731–736 (1979); received December 10, 1978

A modified version of Chandrasekhar's equation is derived with the aid of Karman's energy transfer process for the decay of hydromagnetic turbulence in the limiting case of zero viscosity and infinite electrical conductivity. It is found that the asymptotic behaviour of self-preserving solutions of the aforesaid equations leads to $F(k,t) \approx k^4$, $(c=2/7)$, $G(k,t) \approx k^6$, $(c=2/9)$ as $k \to 0$ and $F(k,t) \approx k^{-5/3}$, $G(k,t) \approx k^{-5/3}$ for $k \to \infty$.

I. Introduction

There is an increasing interest in the field of hydromagnetic turbulence, largely due to its abundant applications in Geophysics and Astrophysics.

An important development in this context is due to Batchelor [1]. According to him, magnetic energy behaves like the energy which is embedded in the vorticity of eddy turbulence. In this context, it is worthwhile to mention that hydromagnetic problems pertaining to turbulent flow were first studied in detail by Chandrasekhar [2, 3, 4] with the use of different order gauze invariant skew isotropic tensors and vector calculus. Like eddy turbulence, deeper analysis of hydromagnetic turbulence also requires the removal of indeterminacy. In this paper, an attempt is made to effect with elementary tools of operation such a removal of indeterminacy. Certain modifications of Chandrasekhar's hydromagnetic equation [3] are made by use of a form of the magnetic spectrum $G(k,t)$ analogous to the eddy spectrum introduced by Karman [5]. This is analyzed for the limiting case of zero viscosity and infinite electrical conductivity. In Section II, a certain generalization of Heisenberg's decay equation of the energy spectrum [6] pertaining to eddy turbulence is mentioned.

This generalization is firstly due to Karman [5]. Incidentally, Heisenberg [6] suggested similar solutions of equations in the form

$$F(k,t) \sim \frac{1}{\sqrt{t}} f(k\sqrt{t}) . \tag{1a}$$

Sen [7] pointed out that (1a) gives one of a whole bunch of similar solutions and in fact, the bunch can be visualized in the form

$$F(k,t) \sim \frac{1}{k^2 k_0{}^3 t_0{}^2} \left(\frac{t}{t_0}\right)^{3c-2} f\left[\frac{k}{k_0}\left(\frac{t}{t_0}\right)^c\right], \tag{1b}$$

where c is restricted to values $< 2/3$.

When $c = 1/2$, (1b) reduces to the form (1a) given by Heisenberg [6]. But (1b) has got an additional interest in as much as (1b) for $c = 2/7$ gives the required asymptotic behaviour

$$F(k,t) \sim k^4, \quad k \to 0 , \tag{2}$$

due to Lin [8], Batchelor [9] and Obukhov [10] and Karman's [5] spectrum

$$F(k,t) \sim k^{-5/3}, \quad k \to \infty . \tag{3}$$

It is to be mentioned that the decay equation of Heisenberg or that of Karman can be abandoned in favour of another form with the use of Millionshchtikov's quasi-normality hypothesis [11]. This has been worked out first by Reid and Proudman [12] and subsequently generalised by Ghosh [13].

Here an attempt has been made to study the hydromagnetic field as suggested by Ghosh [14] for the eddy turbulence.

II. Mathematical Formulation

The relevant hydromagnetic equations derived by Chandrasekhar are given by

Reprint requests to Dr. G. C. Pramanik. Please order a reprint rather than making your own copy.

0340-4811 / 79 / 0600-0731 $ 0.100/0

$$\frac{1}{2}\frac{\partial F(k)}{\partial t} = \bar{k}\sqrt{\frac{F(k)}{k^3}}\int_0^k dk'\, F(k')\, k'^2 + \bar{k}\sqrt{\frac{G(k)}{k^3}}\int_0^k dk'\, G(k')\, k'^2$$
$$- \bar{k}\, F(k)\, k^2 \int_k^\infty dk'' \sqrt{\frac{F(k'')}{k''^3}} - \bar{k}\, G(k)\, k^2 \int_k^\infty dk'' \sqrt{\frac{F(k'')}{k''^3}}$$
$$- \bar{k}\, F(k)\, k^2 \int_k^\infty dk'' \sqrt{\frac{G(k'')}{k''^3}} - \nu\, F(k)\, k^2 , \tag{4a}$$

and

$$\frac{1}{2}\frac{\partial G(k)}{\partial t} = \bar{k}\sqrt{\frac{F(k)}{k^3}}\int_0^k dk'\, G(k')\, k'^2 + \bar{k}\sqrt{\frac{G(k)}{k^3}}\int_0^k dk'\, F(k')\, k'^2$$
$$- \bar{k}\, G(k)\, k^2 \int_k^\infty dk'' \sqrt{\frac{G(k'')}{k''^3}} - \lambda\, G(k)\, k^2 . \tag{4b}$$

In view of the modifications suggested here, these become

$$\frac{1}{2}\frac{\partial F(k,t)}{\partial t} = \bar{k}\Bigg[F^{\alpha}(k,t)\, k^{\beta} \int_0^k dk'\, F^{3/2-\alpha}(k',t)\, k'^{1/2-\beta} - F^{3/2-\alpha}(k,t)\, k^{1/2-\beta} \int_k^\infty dk''\, F^{\alpha}(k'',t)\, k''^{\beta}$$
$$+ G^{\alpha}(k,t)\, k^{\beta} \int_0^k dk'\, G^{3/2-\alpha}(k',t)\, k'^{1/2-\beta} - \Big\{G^{3/2-\alpha}(k,t)\, k^{1/2-\beta} \int_k^\infty dk''\, F^{\alpha}(k'',t)\, k''^{\beta}$$
$$+ F^{3/2-\alpha}(k,t)\, k^{1/2-\beta} \int_k^\infty dk''\, G^{\alpha}(k'',t)\, k''^{\beta}\Big\}\Bigg] - \nu\, k^2\, F(k,t) , \tag{5a}$$

and

$$\frac{1}{2}\frac{\partial G(k,t)}{\partial t} = \bar{k}\Bigg[F^{\alpha}(k,t)\, k^{\beta} \int_0^k dk'\, G^{3/2-\alpha}(k',t)\, k'^{1/2-\beta} + G^{\alpha}(k,t)\, k^{\beta} \int_0^k dk'\, F^{3/2-\alpha}(k',t)\, k'^{1/2-\beta}$$
$$- G^{3/2-\alpha}(k,t)\, k^{1/2-\beta} \int_k^\infty dk''\, G^{\alpha}(k'',t)\, k''^{\beta}\Bigg] - \lambda\, k^2\, G(k,t) , \tag{5b}$$

where $F(k,t)$ and $G(k,t)$ denote the spectrum functions of the turbulent fluid and turbulent magnetic field, respectively. In Eqs. (5a) and (5b) μ is the magnetic permeability, σ the electrical conductivity, ν the kinematic viscosity, λ the magnetic diffusivity and $\bar{k}$ a constant, while $\lambda = \frac{1}{4}\pi\mu\sigma$ and dimensional analysis requires

$$\alpha + \alpha' = \tfrac{3}{2} \quad \text{and} \quad \beta + \beta' = \tfrac{1}{2} . \tag{6}$$

The decay equation can be obtained from Eqs. (5a) and (5b) as

$$-\frac{1}{2}\frac{\partial}{\partial t}\int_0^k [F(k',t) + G(k',t)]\, dk' = \bar{k}\int_0^k dk' \{F^{3/2-\alpha}(k',t)\, k'^{1/2-\beta} + G^{3/2-\alpha}(k',t)\, k'^{1/2-\beta}\}$$
$$\times \int_k^\infty \{F^{\alpha}(k'',t)\, k''^{\beta} + G^{\alpha}(k'',t)\, k''^{\beta}\}\, dk'' + \int_0^k dk' \{\nu\, k'^2\, F(k',t) + \lambda\, k'^2\, G(k',t)\} . \tag{7}$$

a) Self-preserving Solution of the Hydromagnetic Decay Equation

We introduce the self-preserving solutions of Eq. (7) in a very general form as

$$F(k,t) = \frac{1}{\bar{k}^2\, k_0^3\, t_0^2} \cdot \frac{u^q(\bar{\lambda})\, S^{p+1}}{\tau^r} f(\bar{\lambda}\, S) \tag{8a}$$

and

$$G(k,t) = \frac{1}{\bar{k}^2 k_0^3 t_0^2} \cdot \frac{u^{q'}(\bar{\lambda})\, S^{p'+1}}{\tau^{r'}}\, g(\bar{\lambda} S)\,, \tag{8b}$$

where $k_0, t_0, \bar{k}, p, p', q, q', r$ and r' are all constants, $S = S(\tau)$, $\tau = t/t_0$ and $\bar{\lambda} = k/k_0$.

Obviously, $F(k,t)$ and $G(k,t)$ satisfy the equation

$$\begin{aligned}
&\frac{S^p}{\tau^{r+1}}\left[\int_0^x u^q\left(r - \frac{p_\tau S_\tau}{S}\right) f(x')\,\mathrm{d}x' - u^q \frac{\tau S_\tau}{S} x f(x)\right]\\
&\quad + \frac{S^{p'}}{\tau^{r'+1}}\left[\int_0^x u^{q'}\left(r' - \frac{p_\tau' S_\tau}{S}\right) g(x')\,\mathrm{d}x' - u^{q'} \frac{\tau S_\tau}{S} x g(x)\right]\\
&= \frac{2 S^{(p+1)\alpha-\beta-1}}{\tau^{r\alpha}} \int_x^\infty u^{q\alpha} f^\alpha(x')\, x'^\beta \,\mathrm{d}x' \,\frac{S^{p(3/2-\alpha)-\alpha+\beta}}{\tau^{(3/2-\alpha)r}} \int_0^x u^{q(3/2-\alpha)} f^{(3/2-\alpha)}(x')\, x'^{(1/2-\beta)}\,\mathrm{d}x'\\
&\quad + \frac{2 S^{(p+1)\alpha-\beta-1}}{\tau^{r\alpha}} \int_x^\infty u^{q\alpha} f^\alpha(x')\, x'^\beta \,\mathrm{d}x' \,\frac{S^{p'(3/2-\alpha)-\alpha+\beta}}{\tau^{(3/2-\alpha)r'}} \int_0^x u^{q'(3/2-\alpha)} g^{(3/2-\alpha)}(x')\, x'^{(1/2-\beta)}\,\mathrm{d}x'\\
&\quad + \frac{2 S^{(p'+1)\alpha-\beta-1}}{\tau^{r'\alpha}} \int_x^\infty u^{q'\alpha} g^\alpha(x')\, x'^\beta \,\mathrm{d}x' \,\frac{S^{p(3/2-\alpha)-\alpha+\beta}}{\tau^{(3/2-\alpha)r}} \int_0^x u^{q(3/2-\alpha)} f^{(3/2-\alpha)}(x')\, x'^{(1/2-\beta)}\,\mathrm{d}x'\\
&\quad + \frac{2 S^{(p'+1)\alpha-\beta-1}}{\tau^{r'\alpha}} \int_x^\infty u^{q'\alpha} g^\alpha(x')\, x'^\beta \,\mathrm{d}x' \,\frac{S^{p'(3/2-\alpha)-\alpha+\beta}}{\tau^{(3/2-\alpha)r'}} \int_0^x u^{q'(3/2-\alpha)} g^{(3/2-\alpha)}(x')\, x'^{(1/2-\beta)}\,\mathrm{d}x'\\
&\quad + \frac{S^p}{\tau^{r+1}} \cdot \frac{2\tau}{S^2 R} \int_0^x u^q x'^2 f(x')\,\mathrm{d}x' + \frac{S^{p'}}{\tau^{r'+1}} \cdot \frac{2\tau}{S^2 R_m} \int_0^x u^{q'} x'^2 g(x')\,\mathrm{d}x'\,,
\end{aligned} \tag{9}$$

where $\nu k_0^2 t_0 = 1/R$, $\lambda k_0^2 t_0 = 1/R_m$ and $(k/k_0)\, S = x$.

Let us consider the state of motion when dissipation of energy either by viscosity in the form of molecular motion or by conductivity in the form of Joule heat is negligible and transfer of energy by inertial forces is the dominant process. Under the above assumptions, when both eddy Reynolds number R and magnetic Reynolds number R_m are large enough, Eq. (9) reduces to the form

$$\begin{aligned}
&\frac{S^p}{\tau^{r+1}}\left[\int_0^x u^q\left(r - \frac{p_\tau S_\tau}{S}\right) f(x')\,\mathrm{d}x' - u^q \frac{\tau S_\tau}{S} x f(x)\right]\\
&\quad + \frac{S^{p'}}{\tau^{r'+1}}\left[\int_0^x u^{q'}\left(r' - \frac{p_\tau' S_\tau}{S}\right) g(x')\,\mathrm{d}x' - u^{q'} \frac{\tau S_\tau}{S} x g(x)\right]\\
&= \frac{2 S^{(p+1)\alpha-\beta-1}}{\tau^{r\alpha}} \int_x^\infty u^{q\alpha} f^\alpha(x')\, x'^\beta \,\mathrm{d}x' \,\frac{S^{p(3/2-\alpha)-\alpha+\beta}}{\tau^{(3/2-\alpha)r}} \int_0^x u^{q(3/2-\alpha)} f^{(3/2-\alpha)}(x')\, x'^{(1/2-\beta)}\,\mathrm{d}x'\\
&\quad + \frac{2 S^{(p+1)\alpha-\beta-1}}{\tau^{r\alpha}} \int_x^\infty u^{q\alpha} f^\alpha(x')\, x'^\beta \,\mathrm{d}x' \,\frac{S^{p'(3/2-\alpha)-\alpha+\beta}}{\tau^{(3/2-\alpha)r'}} \int_0^x u^{q'(3/2-\alpha)} g^{(3/2-\alpha)}(x')\, x'^{(1/2-\beta)}\,\mathrm{d}x'\\
&\quad + \frac{2 S^{(p'+1)\alpha-\beta-1}}{\tau^{r'\alpha}} \int_x^\infty u^{q'\alpha} g^\alpha(x')\, x'^\beta \,\mathrm{d}x' \,\frac{S^{p(3/2-\alpha)-\alpha+\beta}}{\tau^{(3/2-\alpha)r}} \int_0^x u^{q(3/2-\alpha)} f^{(3/2-\alpha)}(x')\, x'^{(1/2-\beta)}\,\mathrm{d}x'\\
&\quad + \frac{2 S^{(p'+1)\alpha-\beta-1}}{\tau^{r'\alpha}} \int_x^\infty u^{q'\alpha} g^\alpha(x')\, x'^\beta \,\mathrm{d}x' \,\frac{S^{p'(3/2-\alpha)-\alpha+\beta}}{\tau^{(3/2-\alpha)r'}} \int_0^x u^{q'(3/2-\alpha)} g^{(3/2-\alpha)}(x')\, x'^{(1/2-\beta)}\,\mathrm{d}x'\,.
\end{aligned} \tag{10}$$

Here the conditions of similarity will be satisfied if

$$p = p' = 2, \quad q = q' = 0\,, \quad s = \alpha_1 \tau^c\,, \quad r = r'\,, \tag{11}$$

and $r = c(p-2)+2$, where α_1 is the constant of integration of the differential equation,

$$\tau S_\tau / S = c$$

and c is another constant.

Substituting the similarity conditions (11) in Eq. (10) we obtain

$$2(1-c)\int_0^x \{f(x')+g(x')\}\,\mathrm{d}x' - c\,x\{f(x)+g(x)\}$$
$$= 2\int_0^x \{f^{3/2-\alpha}(x')\,x'^{1/2-\beta}+g^{3/2-\alpha}(x')\,x'^{1/2-\beta}\}\,\mathrm{d}x' \int_x^\infty \{f^\alpha(x')\,x'^\beta+g^\alpha(x')\,x'^\beta\}\,\mathrm{d}x'\,. \tag{12}$$

III. Asymptotic Behaviour of $f(x)$ and $g(x)$ for $x \to 0$ and $x \to \infty$

a) Asymptotic Behaviour of $f(x)$ and $g(x)$ for $x \to 0$

For $x \to 0$, $f(x')$ and $g(x')$ may be expanded as

$$f(x') = f(x) + (x'-x)\,f'(x) + \cdots, \quad g(x') = g(x) + (x'-x)\,g'(x) + \cdots,$$

which gives

$$\int_0^x x'^{1/2-\beta} f^{3/2-\alpha}(x')\,\mathrm{d}x' \sim \frac{f^{3/2-\alpha}(x)\,x^{3/2-\beta}}{(\frac{3}{2}-\beta)} \tag{13}$$

and

$$\int_0^x x'^{1/2-\beta} g^{3/2-\alpha}(x')\,\mathrm{d}x' \sim \frac{g^{3/2-\alpha}(x)\,x^{3/2-\beta}}{(\frac{3}{2}-\beta)}\,. \tag{14}$$

Differentiating Eq. (12) and using Eqs. (13) and (14), we obtain after simplification

$$[(2-4c)\,Y + c\,x\,Y'][f'(x)+g'(x)] - (2-3c)\,Y'[f'(x)+g'(x)] - c\,x\,Y[f''(x)+g''(x)]$$
$$= -\frac{2}{(\frac{3}{2}-\beta)}[\alpha\,f^{\alpha-1}(x)\,f'(x)\,r^{\beta+1} + (\beta+1)\,f^\alpha(x)\,x^\beta + \alpha\,g^{\alpha-1}(x)\,g'(x)\,x^{\beta+1}$$
$$+ (\beta+1)\,g^\alpha(x)\,x^\beta]\,Y^2 - 2\,[f^\alpha(x)\,x^\beta + g^\alpha(x)\,x^\beta]\,Y^2, \tag{15}$$

where $Y = [f^{3/2-\alpha}(x)\,x^{1/2-\beta} + g^{3/2-\alpha}(x)\,x^{1/2-\beta}]$ and $f'(x) \equiv \mathrm{d}f/\mathrm{d}x$, $g'(x) \equiv \mathrm{d}g/\mathrm{d}x$ etc. If we substitute

$$f(x) \sim A\,x^n, \quad A \neq 0, \quad x \to 0 \quad \text{and} \quad g(x) \sim B\,x^{n'}, \quad B \neq 0, \quad x \to 0\,,$$

in Eq. (15), one finds

$$[\{n - n(\tfrac{3}{2}-\alpha) - (\tfrac{1}{2}-\beta)\}(2-3c) - c\,n^2 + c\,n^2(\tfrac{3}{2}-\alpha) + c\,n(\tfrac{1}{2}-\beta)]\,A^{5/2-\alpha}\,x^{(5n-1)/2-n\alpha-\beta}$$
$$+ [\{n' - n'(\tfrac{3}{2}-\alpha) - (\tfrac{1}{2}-\beta)\}(2-3c) - c\,n'^2 + c\,n'^2(\tfrac{3}{2}-\alpha) + c\,n'(\tfrac{1}{2}-\beta)]\,B^{5/2-\alpha}\,x^{(5n'-1)/2-n'\alpha-\beta}$$
$$+ [\{n' - n(\tfrac{3}{2}-\alpha) - (\tfrac{1}{2}-\beta)\}(2-3c) - c\,n'^2 + c\,n\,n'(\tfrac{3}{2}-\alpha) + c\,n'(\tfrac{1}{2}-\beta)]\,A B^{3/2-\alpha}\,x^{(3n-1)/2-n\alpha+n'-\beta}$$
$$+ [\{n - n'(\tfrac{3}{2}-\alpha) - (\tfrac{1}{2}-\beta)\}(2-3c) - c\,n^2 + c\,n\,n'(\tfrac{3}{2}-\alpha) + c\,n(\tfrac{1}{2}-\beta)]\,A B^{3/2-\alpha}\,x^{(3n'-1)/2-n'\alpha+n-\beta}$$
$$+ \left[\frac{\alpha\,n+(\beta+1)}{\frac{3}{2}-\beta}+1\right] 2\,A^{3-\alpha}\,x^{(3-\alpha)n+1-\beta} + \left[\frac{\alpha\,n'+\beta+1}{(\frac{3}{2}-\beta)}+1\right] 2\,B^{3-\alpha}\,x^{(3-\alpha)n'+1-\beta}$$
$$+ \left[\frac{\alpha\,n+\beta+1}{\frac{3}{2}-\beta}+1\right] 2\,A^\alpha\,B^{3-2\alpha}\,x^{(3-2\alpha)n'+\alpha n+1-\beta} \tag{16}$$
$$+ \left[\frac{\alpha\,n'+\beta+1}{\frac{3}{2}-\beta}+1\right] 2\,B^\alpha\,A^{3-2\alpha}\,x^{(3-2\alpha)n+\alpha n'+1-\beta}$$
$$+ \left[\frac{\alpha\,n+\beta+1}{\frac{3}{2}-\beta}+1\right] 4\,A^{3/2}\,B^{3/2-\alpha}\,x^{(3/2-\alpha)n'+3/2n+1-\beta}$$
$$+ \left[\frac{\alpha\,n'+\beta+1}{\frac{3}{2}-\beta}+1\right] 4\,B^{3/2}\,A^{3/2-\alpha}\,x^{(3/2-\alpha)n+3/2n'+1-\beta}\,.$$

In order to find the energy spectrum we consider the following cases:

a) When $n<n'$, the first term in Eq. (16) is significant. Therefore, equating the coefficients of $x^{(5n-1)/2-n\alpha-\beta}$ to zero, we get two values of n as

$$n_1 = \frac{2-3c}{c}, \tag{17}$$

$$n_2 = \frac{2\beta-1}{1-2\alpha}. \tag{18}$$

As $x \to 0$, $f(x) \sim A\, x^{(2-3c)/c}$ for which

$$F(k,t) \sim \text{constant} \cdot k^{(2-3c)/c} \quad \text{for} \quad k \to 0,$$

and it is to be noted that

$$F(k,t) \sim \text{constant} \cdot k^4 \quad \text{for} \quad c = 2/7,$$

which was obtained by Sen [7] in absence of magnetic field.

On the other hand, $n_2 > 0$ has two possibilities:

(i) $\alpha > \frac{1}{2}$, $\beta < \frac{1}{2}$, (ii) $\alpha < \frac{1}{2}$, $\beta > \frac{1}{2}$.

But for the case (i), $F(k,t)$ begins with K^4 for $k \to 0$ provided $4\alpha+\beta<5/2$, and if

$$4\alpha+\beta > 5/2, \quad f(x) \sim \text{constant} \cdot x^{n_2},$$

($n_2 < 4$) as $x \to 0$. But for the case (ii) one obtains the reverse result and Karman's condition [5] is applicable only for the first case.

(b) If $n' < n$, then proceeding in a manner similar to the case (a), we get

$$n_1' = \frac{2-3c}{c}$$

and $$n_2' = \frac{2\beta-1}{1-2\alpha} \quad \text{for} \quad x \to 0,$$

where $c < \frac{2}{3}$.

When

$$n_1' = \frac{2-3c}{c},$$

$$G(k,t) \sim \text{constant} \cdot k^6, \quad (k \to 0),$$

(Chandrasekhar [2]) with $c = 2/q$ and $|\overline{h^2}|$ behaves like $|\overline{w^2}|$ (Batchelor [1]).

The behaviour of n_2' will be the same as in case a).

b) Asymptotic Behaviour of $f(x)$ and $g(x)$ for $x \to \infty$

In order to obtain the behaviour of $f(x)$ and $g(x)$ for $x \to \infty$, we substitute $f(x') = e^{-\Omega_1(x')}$ and $g(x') = e^{-\Omega_2(x')}$ in Eq. (12) and then, following the Heisenberg's methods of approximation, we get

$$\begin{aligned}
&\frac{(2-3c)[f(x)+g(x)] - c\,x[f'(x)+g'(x)]}{f^\alpha(x)\,x^\beta + g^\alpha(x)\,x^\beta} \\
&= -2\int_0^x [f^{3/2-\alpha}(x')\,x'^{1/2-\beta} + g^{3/2-\alpha}(x')\,x'^{1/2-\beta}]\,dx' \\
&\quad -2\left[\frac{f^\alpha(x)\,x^{\beta+1}}{\beta+1+\alpha x\dfrac{f'(x)}{f(x)}} + \frac{g^\alpha(x)\,x^{\beta+1}}{\beta+1+\alpha x\dfrac{g'(x)}{g(x)}}\right] \\
&\quad \cdot \frac{[f^{3/2-\alpha}(x)\,x^{1/2-\beta} + g^{3/2-\alpha}(x)\,x^{1/2-\beta}]}{[f^\alpha(x)\,x^\beta + g^\alpha(x)\,x^\beta]},
\end{aligned} \tag{19}$$

provided $\alpha\Omega_1' - \beta - 1 > 0$ and $\alpha\Omega_2' - \beta - 1 > 0$.

We assume solutions of the Eq. (19) in the forms

$$f(x) \sim c_1 x^{-n}, \quad (x \to \infty), \quad c_1 \neq 0,$$

and

$$g(x) \sim \mathrm{d}x^{-n'}, \quad (x \to \infty), \quad d \neq 0, \tag{20}$$

where n and n' are positive numbers. The Eq. (19) becomes, by using (20):

$$\begin{aligned}
&(2-3c+c\,n)\,c_1 x^{-n} + (2-3c+c\,n')\,\mathrm{d}x^{-n'} \\
&+2\left(\frac{1}{A_1}+\frac{1}{M}\right)c_1^{3/2}\,x^{(-3n+3)/2} \\
&+2\left(\frac{1}{B_1}+\frac{1}{N}\right)\mathrm{d}^{3/2}x^{(-3n'+3)/2} \\
&+2\left(\frac{1}{A_1}+\frac{1}{N}\right)c_1^{3/2-\alpha}\,\mathrm{d}^\alpha\,x^{(-3n+2n\alpha-2n'\alpha+3)/2} \\
&+2\left(\frac{1}{B_1}+\frac{1}{M}\right)\mathrm{d}^{3/2-\alpha}\,c_1^\alpha\,x^{(-3n'+2n'\alpha-2n\alpha+3)/2} = 0,
\end{aligned} \tag{21}$$

where

$$A_1 = \frac{3-3n+2n\alpha-2\beta}{2},$$

$$B_1 = \frac{3-3n'+2n'\alpha-2\beta}{2},$$

$$M = \beta+1-\alpha n,$$

$$N = \beta+1-\alpha n'.$$

In order to obtain the behaviour of $f(x)$ and $g(x)$ for $x \to \infty$, we consider the following cases:

(a) When $n<n'$, we get $n=5/3$, which leads to

$$F(k,t) \sim \text{constant} \cdot k^{-5/3} \quad \text{for} \quad k \to \infty,$$

(b) if $n'<n$, we obtain $n'=5/3$ giving

$$G(k,t) \sim \text{constant} \cdot k^{-5/3} \quad \text{for} \quad k \to \infty.$$

Therefore, as the small scale motion ($x \to \infty$ i.e. $k \to \infty$) [15] of the hydromagnetic turbulence is appeared, the spectrum of both eddy turbulence and magnetic turbulence attain the well-known $-5/3$ power law of Obukhov-Kolmogoroff.

(c) It is well known that the kinetic energy of the turbulent velocity field stretches the magnetic lines of force and thereby increases the strength of the magnetic field till equipartition of energy is reached between the magnetic and eddy parts of energy. Therefore, when an equipartition between these two forms of energy is attained, one may set $n = n'$, for which Eq. (21) is replaced by

$$(2 - 3c + c\,n)(c_1 + d)\,x^{-n} + 2\left(\frac{1}{A_1} + \frac{1}{M}\right) \cdot (c_1{}^{3/2} + d^{3/2} + c_1{}^{3/2-\alpha}\,d^{\alpha} + c_1{}^{\alpha}\,d^{3/2-\alpha}) \cdot x^{(-3n+3)/2} = 0. \quad (22)$$

If the first term of Eq. (22) is significant, i.e. $n < (3n-3)/2$, we get $n = 3 - 2/c$. But this violates the restriction $c < \frac{2}{3}$. Hence this is not admissible.

On the other hand, when $n > (3n-3)/2$, the coefficient of $x^{(-3n+3)/2}$ leads to $n = 5/3$, giving the Obukhov-Kolmogoroff power law.

Thus we find that the spectra of both eddy turbulence and magnetic turbulence obey the Obukhov-Kolmogoroff law irrespective of the existence or non-existence of equipartition of two forms of energy.

Acknowledgement

The author is indebted to Prof. K. M. Ghosh for his kind interest in the work.

[1] G. K. Batchelor, Proc. Roy. Soc. London **A 201**, 405 (1950).
[2] S. Chandrasekhar, Proc. Roy. Soc. London **A 207**, 301 (1951).
[3] S. Chandrasekhar, Proc. Roy. Soc. London **A 233**, 330 (1955).
[4] S. Chandrasekhar, Ann. Phys. N.Y. **2**, 615 (1957).
[5] T. V. Karman, Proc. Nat. Acad. Sci. **34**, 530 (1948).
[6] W. Heisenberg, Proc. Roy. Soc. London **A 195**, 402 (1948).
[7] N. R. Sen, Bull. Cal. Math. Soc. **43**, 1 (1951).
[8] C. C. Lin, First Symposium of Applied Mathematics, Amer. Math. Soc. (1947).
[9] G. K. Batchelor, Proc. Roy. Soc. London **A 195**, 513 (1949).
[10] A. M. Obukhov, Comp. Rend. Acad. Sci. U.R.S.S. **32**, 19 (1941).
[11] M. Millionshchtikov, C.R. Acad. Sci. U.R.S.S. **32**, 615 (1941).
[12] I. Proudman and W. H. Reid, Phil. Trans. Roy. Soc. London **A 247**, 163 (1954).
[13] K. M. Ghosh, Indian J. Pure Appl. Math. 3, 157 (1972).
[14] K. M. Ghosh, Proc. Nat. Inst. Sci. India **XX**, 336 (1954).
[15] Y. Paul and S. R. Patel, Appl. Sci. Res. **33**, 187 (1977).